# Developing a framework for viewing affective and knowledge factors in teaching primary mathematics

**Jean Carroll**

**Royal Melbourne Institute of Technology**

An extensive study of the literature on teachers' views of mathematics and mathematics teaching revealed inadequacies in the available frameworks for structuring and reporting on research. In this paper a new framework is developed for analysing and describing primary school teachers' knowledge and feelings about mathematics teaching and learning. The framework; 'Teacher Type Table', was developed after surveying 107 primary school teachers in suburban Melbourne. The teacher type table is presented here as a model for use in the professional development of teachers.

## Introduction

The research which is reported in this paper arose in response to well publicised concerns about the knowledge and attitudes of primary school teachers with respect to teaching mathematics. This paper reports on some qualitative components of a study (Carroll, 1997) which was designed to investigate how primary school teachers view themselves as teachers of mathematics. The paper identifies issues from research on primary teachers and their mathematics teaching, analyses the results of a survey of primary school teachersí views and presents a new framework for analysing and describing teachersí knowledge and affective factors regarding their mathematics learning and teaching.

## Issues in the Review of Research

An extensive review of the research literature on affective and knowledge factors in teaching primary school mathematics (Carroll, 1997), revealed a number of important issues which are discussed below.

### *Affective and Knowledge Factors of Primary Teachers*

The research review supported the view that primary teachers *were* limited in their knowledge of mathematics (Cockroft, 1982, p.189; Fennema & Franke, 1992; Kanes & Nisbet, 1994). Beginning and preservice primary school teachersí knowledge was also found to be lacking in their understanding of methods for representing mathematical ideas in ways that are understandable for students (Ball, 1990, as cited in Fennema & Franke, 1992, p.154; Sullivan,1989, p.15).

Many studies reported that negative attitudes towards mathematics were held by primary school teachers and teacher trainees (Ball 1988, Bobis & Cusworth, 1994; Carroll, 1994; Cockroft, 1982, para.679; Hart, 1993; Relich Way & Martin, 1994, p.58; Sullivan, 1987). Researchers expressed their concerns about the high proportions of teacher trainees who are lacking in confidence in their ability to do mathematics and their knowledge of how to teach mathematics to children. The changes in attitudes and beliefs of trainee teachers during their training were well documented.

### *Lack of Participation by Classroom Teachers in Research Studies*

Less well understood and studied, however, was the of evolution affective and knowledge factors when teacher training is completed and a teacher begins to teach in the classroom. A number of studies reported on knowledge and affective factors of

primary school teachers who were undertaking further study or who were involved in professional development programmes focussed on their teaching of mathematics (Cobb, Yackel & Wood, 1992; Fennema, Carpenter, Franke, Levi, Jacobs & Empson, 1996). Research which reported on the knowledge and affective factors related to teachers working in primary schools, who were not pursuing some form of further study was limited and signalled a need for research to be conducted that examines the views of primary teachers across the board, rather than just those training, beginning or returning to study.

*Inadequacy of Models for Researching Teachersí Knowledge and Affective Factors*

A further issue concerned the inadequacy of available frameworks for structuring and reporting research on the mathematics teaching and learning. A number of models related to teaching and learning were considered in the development of the present study and while useful in constructing a theoretical framework for reporting on the literature, designing the study and reporting results, these models were all found to be limited.

Koehler and Grouws (1992, p.117) presented a model for researching the mathematics teaching and learning process which involved a range factors including teachersí knowledge of content, pedagogy and student learning, and teachersí attitudes towards and beliefs about teaching and mathematics. Their model provided a way of structuring discussion and presenting research, however it failed to reflect the dynamic and interactive nature of the process of teaching and learning mathematics and also omitted an important component of the teaching process; the ëteacherís knowledge of mathematical representationsí (Ball, 1988; Shulman, 1987, p. 8; Bromme, 1994, p. 75).

McLeod (1992) in his comprehensive review of the research on affective factors, criticised many studies of mathematics teaching and learning which in the past had focussed on knowledge and tended to ignore affective factors. McLeod (1989b) had earlier attempted to redress this neglect when he proposed a model which divided the affective domain into beliefs, attitudes and emotions. His model showed relationships between beliefs, attitudes and emotions and cognitive involvement and also incorporated the notion that affective factors are changeable and linked to cognitive factors. The ideas presented in McLeodís model attempt to reflect the dynamic, interactive nature of cognitive and affective factors in mathematics education in ways which were lacking in the Koehler and Grouws (1992) model.

Examination of the limitations of some key theoretical models for research into the teaching of mathematics suggests the need for new frameworks to be developed in order to better understand the complexity of these factors and to design further study in the area.

## Investigating Primary Teachersí Views of Mathematics Teaching

The study reported here was designed to address the question of how primary school teachers view their; knowledge of mathematics, knowledge of the approaches required for teaching mathematics, attitudes and beliefs about doing mathematics, and attitudes and beliefs about teaching mathematics

*Method*: The data was collected using a survey instrument; the ëMathematics Attitude and Knowledge Scaleí (MAK scale) which was designed to investigate how primary teachers felt about teaching mathematics, how they rated their knowledge of

mathematics and the approaches required for teaching it and also how they felt about their own learning of mathematics and their understanding of it.

The scale was developed by identifying five constructs relevant to the literature on teachers' views of mathematics and mathematics teaching (Nisbet, 1991, p.37; Relich, 1995). The five constructs were:

1. knowledge, competence and understanding of mathematics;
2. knowledge, competence and understanding of mathematics teaching;
3. enjoyment and confidence in learning and doing mathematics;
4. enjoyment and confidence in teaching mathematics and
5. conceptions of mathematics and mathematics teaching.

Six items were used for each construct; three items were worded positively (for example, ìI have always done well in mathematics classesî) and three worded negatively (for example, ìIím not very good at mathematicsî). Some of the items were adapted from scales developed by Nisbet (1991) and others were adapted from a scale developed by Relich. (1995). The MAK scale consisted of 30 items requiring responses on a five point Likert-type scale ranging through strongly agree, agree, not sure, disagree to strongly disagree. Table 1 lists the thirty items and the five constructs are identified in column 3. The positive and negative signs in column 3 of the table identify whether the item was positively or negatively worded. The scale was also designed to collect biographical information.

*Procedure*: Twenty Department of Education primary schools were chosen at random for the study from the South Central Metropolitan Region of Melbourne. Permission to conduct the research was sought from school principals and received from 13 of the schools. Surveys were delivered to each classroom teacher in the schools and collected several weeks later.

*Analysis of data*

*Coding of responses:* The teachers responded to the MAK scale items using the five point scale; strongly agree, agree, not sure, disagree to strongly disagree. For positively worded items, responses were coded: strongly agree, 5; agree, 4; not sure, 3; disagree 2; and strongly disagree, 1. The negatively worded items were coded: strongly agree, 1; agree, 2; not sure, 3; disagree 4; and strongly disagree, 5. The teachersí coded responses were analysed using the SPSS statistical package.

*Principal Components and Factor analysis:* The MAK scale had been constructed with five scales which were based on the model proposed by Koehler and Grouws. Principal components and factor (Oblimin rotation) analyses were used to interrogate these scales; investigating and summarising patterns of correlations among the items and consequently testing the theoretical framework on which the original five scales were based.

*Factor Scores:* The factors identified by the principal components and factors analysis were used as a basis for the computation of factor scores for each teacher on each of the factors. The factor scores were computed by summing the teachersí coded scores for each item in a factor. A mean score was computed for each factor. An individual teacherís factor score was considered to indicate a negative view on a factor (negative factor tendency) if it was below the mean on that factor. A positive view on a factor (positive factor tendency) was indicated by a factor score above the mean.

Table 1. *Items Loading on 4 Factors*

| Factor 1 (**F**- **F**eelings about teaching mathematics) | | |
|---|---|---|
| No. | ITEM | Construct |
| 24 * | I look forward to teaching mathematics | 4 + |
| 19 * | I do not enjoy teaching mathematics | 4 - |
| 4 * | I enjoy teaching mathematics to primary school children | 4 + |
| 14 * | I find teaching mathematics challenging and exciting | 4 + |
| 29 * | I do not feel confident about teaching mathematics | 4 - |
| 12 * | Teaching mathematics doesn't scare me at all | 2+ |
| 26 * | Wherever possible I avoid doing mathematics | 1 - |
| Factor 2 (**M**- Knowledge of **M**athematics and feelings about doing it) | | |
| 21 * | At school my friends came to me for help in mathematics | 1 + |
| 1 * | I have always done well in mathematics classes | 1 + |
| 16 * | I'm not very good at mathematics | 1 - |
| 11 * | I have generally done better in mathematics courses than other subjects courses | 1 + |
| 3 * | I find many maths problems interesting and challenging | 3 + |
| 13 * | I enjoy doing mathematical problems | 3 + |
| 8 * | I feel frustrated doing mathematical problems | 3 - |
| 9 * | I get a sinking feeling when a child asks a hard question when I'm teaching mathematics | 4 - |
| Factor 3 (**K**- **K**nowledge of methods and approaches for teaching mathematics)) | | |
| 22 | I am confident about my knowledge of current approaches to teaching | 2 + |
| 7 * | I feel that I need to know more about the methods of teaching mathematics to primary school children | 2 - |
| 27 * | I worry that I'm not using the right teaching methods in mathematics | 2- |
| 17* | I'm not sure about what to do when teaching mathematics to primary school children | 2 - |
| Factor 4 (Conceptions of mathematics and mathematics teaching) | | |
| 20 | Maths is mostly facts and procedures which need to be memorised | 5 - |
| 15 | Each student should be encouraged to build on their own maths ideas even if there is much trial and error. | 5 + |
| 30 | Discovery methods of teaching tend to frustrate many students who make too many errors before making any discoveries | 5 - |
| 25 | There are many ways to solve a maths problem | 5 + |
| 10 | Solving a maths problem usually involves finding a rule or formula that applies | 5 - |
| The following questions did not have loadings of > .48 on any factor | | |
| 2 * | I feel secure about teaching mathematics | 2 + |
| 18 * | Mathematics makes me feel inadequate | 3 - |
| 23 | Doing maths makes me feel clever | 3 + |
| 28 * | When I think of doing mathematics, I remember how bad I felt about doing maths tests. | 3 - |
| 5 | Studying mathematics helps to develop the ability to think creatively | 5 + |
| 6 * | I have hesitated to take courses in mathematics | 1 - |

*Results*

One hundred and seven primary school teachers completed and returned the Mathematic Attitude and Knowledge scale; 95 were female and 12 were male. The teachersí experience ranged from first year out to 32 years teaching, with the average being 14 years in the classroom.

*Factor Analysis:* The factor analysis using Oblimin rotation (Carroll, 1997, p.87) identified four factors which had items with loadings greater than 0.48. Table 1 lists the 30 items which were contained in the MAK scale and items which load significantly on each of the four factors.

The first factor contained items relating to the teachers' feelings about teaching mathematics. The second factor consisted of items related to the teachers' views of their knowledge of mathematics and their feelings about doing it. Items which addressed the teachers' knowledge of the methods and approaches required to teach mathematics to primary school children were clustered in the third factor while the fourth factor related to the conceptions of mathematics.

*Principal components analysis:* The principal components factor matrix showed that 22 items (asterisked in Table 1) loaded substantially (loadings greater than 0.5) on one factor. The 22 items all related to the teachersí knowledge and feelings about mathematics or mathematics teaching. This could be considered to represent the underlying construct for these items. The remaining items related mainly to the teachersí views of what mathematics is and how it should be taught.

The factor solution generated by factor analysis and the underlying construct identified in the principal components analysis were used to select the three factors which generated the Teacher Type Table. It can be seen in Table 1 that the items contained in factors one, two and three contributed substantially to the principal construct: the teachersí knowledge and feelings about mathematics or mathematics teaching. Items contained in factor 4 in table 1 showed insignificant loading on this construct. Consequently, for the purposes of analysing and discussing the teachersí views of their knowledge and feelings about mathematics or mathematics teaching, only the first three factors were considered in the teacher type table.

## Development of a New Framework: the Teacher Type Table

A new framework for analysing and describing affective and knowledge factors in teaching primary mathematics was developed by considering the teachersí factor scores on the first three factors in Table 1. Teachersí scores which were below the mean on a factor were considered to indicate a negative factor tendency, while scores above the mean represent a positive factor tendency. The symbols; **F+, F-, M+, M-, K+, K-** summarise the factor tendencies. A profile of each teacher was developed by considering their tendencies on each of the factors. Table 2 shows the eight teacher types and percentage of teachers who were in each category.

## Discussion

The Teacher Type Table is a descriptive tool which provides a new perspective on the teacher. It presents a framework for reporting and mapping changes in teachersí views of their knowledge of mathematics and feelings about learning and doing it, and their knowledge of the approaches for teaching mathematics and feelings about

Table 2. *Teacher Type Table*

| | |
|---|---|
| **F-M-K-** *23%*<br>Negative **feelings about teaching mathematics** including lack of confidence, lack of enjoyment and finding it threatening<br>Knowledge and feelings about **doing or studying mathematics** are negative; have not done well at maths, maths is not the best subject and find doing maths problems frustrating.<br>Lacking in **knowledge about the methods and approaches for teaching mathematics to primary school children** | **F+M-K-** *3%*<br>Positive **feelings about teaching mathematics** including confidence, enjoyment, excitement, challenging and find it non threatening.<br>Knowledge and feelings about **doing or studying mathematics** are negative; have not done well at maths, maths is not the best subject and find doing maths problems frustrating.<br>Lacking in **knowledge about the methods and approaches for teaching mathematics to primary school children** |
| **F-M+K-** *12%*<br>Negative **feelings about teaching mathematics** including lack of confidence, lack of enjoyment and finding it threatening<br>Knowledge and feelings about **doing or studying mathematics** are positive; have done well at maths, better in maths than other subjects and find maths problems interesting and challenging<br>Lacking in **knowledge about the methods and approaches for teaching mathematics to primary school children** | **F+M+K-** *8%*<br>Positive **feelings about teaching mathematics** including confidence, enjoyment, excitement, challenging and find it non threatening.<br>Knowledge and feelings about **doing or studying mathematics** are positive; have done well at maths, better in maths than other subjects and find maths problems interesting and challenging<br>Lacking in **knowledge about the methods and approaches for teaching mathematics to primary school children** |
| **F-M-K+** *11%*<br>Negative **feelings about teaching mathematics** including lack of confidence, lack of enjoyment and finding it threatening<br>Knowledge and feelings about **doing or studying mathematics** are negative; have not done well at maths, maths is not the best subject and find doing maths problems frustrating.<br>Knowledgeable about the methods and approaches for teaching mathematics to primary school children. | **F+M-K+** *6%*<br>Positive **feelings about teaching mathematics** including confidence, enjoyment, excitement, challenging and find it non threatening.<br>Knowledge and feelings about **doing or studying mathematics** are negative; have not done well at maths, maths is not the best subject and find doing maths problems frustrating.<br>Knowledgeable about the methods and approaches for teaching mathematics to primary school children. |
| **F-M+K+** *5%*<br>Negative **feelings about teaching mathematics** including lack of confidence, lack of enjoyment and finding it threatening<br>Knowledge and feelings about **doing or studying mathematics** are positive; have done well at maths, better in maths than other subjects and find maths problems interesting and challenging<br>Knowledgeable about the methods and approaches for teaching mathematics to primary school children. | **F+M+K+** *31%*<br>Positive **feelings about teaching mathematics** including confidence, enjoyment, excitement, challenging and find it non threatening.<br>Knowledge and feelings about **doing or studying mathematics** are positive; have done well at maths, better in maths than other subjects and find maths problems interesting and challenging<br>Knowledgeable about the methods and approaches for teaching mathematics to primary school children. |

teaching it. The Mathematics Attitude and Knowledge scale is readily administered and consequently, teacher type profiles can be quickly developed. The Teacher Type Table has potential in the area of professional development. It could be used prior to embarking on professional development in assessing teachersí needs and could also be of use in evaluating the effectiveness of professional development programs by charting changes in teacher type profiles.

The Teacher Type Table addresses the shortcomings of the Koehler and Grouws model discussed earlier in this paper. The Koehler and Grouws model was found

lacking in its ability to represent the interactivity of cognitive and affective factors. It also neglected the teachersí knowledge of mathematical representations. The Teacher Type Table incorporates the notion that knowledge and affective factors are interactive and changing and factor three includes of teachersí knowledge of mathematical representations.

The Teacher Type Table requires further testing to fully investigate its potential. A limited number of mathematical life histories were analysed and tended to support the categorisations in the table, however further validation studies are required. It may be of use to have teachers self select their teacher types after completing the MAK scale to provide a comparison for the teacher types generated by their factor scores. The use of the mean as the cut off point between positive and negative factor tendencies was supported by the life history data, but further studies are needed to fully investigate the cut off points. As can be seen in Table 2, the extreme teacher types; F-M-K- and F+M+K+ were the largest groups, while some of the teacher types were quite small. Further investigation of the MAK scale and Teacher Type Table in conjunction with qualitative studies should provide increased understanding of the categories.

The Teacher Type Table provides a new way of viewing primary mathematics teachersí knowledge and feelings which is based upon their own reports. While further study will consolidate and perhaps provide modifications, The Table has considerable potential in identifying professional development needs.

## Reference List


Ball, D. L. (1988). Unlearning to teach mathematics. *For the Learning of Mathematics, 8* (1), 40-47.

Bobis, J., & Cusworth, R. (1994). Teacher education: A watershed for preservice teachers' attitudes towards mathematics. In G. Bell, B. Wright, N. Leeson & J. Geake (Eds.). *Proceedings of 17th annual conference of the Mathematics Education Research Group of Australasia.* (pp. 113-120). Lismore: MERGA.

Bromme, R. (1994). Beyond subject matter: A psychological topology of teachers' professional knowledge. In R. Biehler, W. Scholz, R. Straber & B. Winkelmann (Eds.). *Didactics of Mathematics as a Scientific Discipline.* (pp. 73-88). Dordrecht, The Netherlands: Kluwer Academic Publishers.

Carroll, J. (1997). *Primary school teachersí views of mathematics teaching and learning.* Unpublished doctoral dissertation. Monash University, Melbourne.

Carroll, J. (1994). What makes a person mathsphobic? A case study investigating affective, cognitive and social aspects of a trainee teacher's mathematical understanding and thinking. *Mathematics Education Research Journal, 6* (2), 131-143

Cobb, P., Yackel, E. & Wood, T. (1992). A constructivist alternative to the representational view of the mind in mathematics education. *Journal for Research in Mathematics Education, 23* (4), 2-33.

Cockroft, W. H. (Chairman) (1982). *Mathematics counts: Report of the Cockroft Committee of Enquiry into the teaching of mathematics in schools.* London: Her Majesty's Stationery Office.

Fennema, E., & Franke, M. L. (1992). Teachers' knowledge and its impact. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 147-164). New York: Macmillan Publishing Company.

Fennema, E., Carpenter, T. P., Franke, M. L., Levi, L., Jacobs, V. R. & Empson, S. B. (1996). A longitudinal study of learning to use children's thinking in mathematics instruction. *Journal for Research in Mathematics Education, 27* (4), 403-434.

Hart, K. (1993). Confidence in success. In I. Hirabayashi, N. Nohda, K. Shigematsu & F, Lin (Eds.), *Proceedings of the 17th International Conference of Psychology of Mathematics Education.* (pp. 17-45) Japan: Program Committee of the Seventeenth International Conference for the Psychology of Mathematics Education.

Kanes, C., & Nisbet, S. (1994). An investigation into the knowledge bases of primary and secondary mathematics teachers: Report on a pilot study. *Proceedings of 17th annual conference of the Mathematics Education Research Group of Australasia.* (pp. 363-372). Lismore.

Koehler, M. S., & Grouws, D. A. (1992). Mathematics teaching practices and their effects. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 115-126). New York: Macmillan Publishing Company.

McLeod, D. B. (1989a). The role of affect in mathematical problem solving. In D. B. McLeod & V. M Adams (Eds.). *Affect and mathematical problem solving* (pp. 20-36). New York: Springer-Verlag.

McLeod, D. B. (1989b). Beliefs, attitudes and emotions: New views of affect in mathematical education. In D. B. McLeod & V. M Adams (Eds.), *Affect and mathematical problem solving* (pp. 245-258). New York: Springer-Verlag.

McLeod, D. B. (1992). Research on affect in mathematics education: A reconceptualisation. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 39-48). New York: Macmillan Publishing Company.

Nisbet, S. (1991). A new instrument to measure pre-service primary teacher's attitudes to teaching mathematics. *Mathematics Education Research Journal, 3* (2), 35-56.

Relich, J. (1995). Gender, self-concept and teachers of mathematics: Effects of attitudes to teaching and learning. *Educational Studies in Mathematics, 30* (2), 179-197.

Relich, J., Way, J., & Martin, A. (1994). Attitudes to teaching mathematics: Further development of a measuring instrument. *Mathematics Education Research Journal. 6* (1), 56-69.

Shulman, L. S. (1987). Knowledge and teaching: Foundations for the new reform. *Harvard Educational Review, 57* (1), 1-22

SPSS for Windows Release 6. Chicago: SPSS Inc.

Sullivan, P. (1987). The impact of a pre-service mathematics education course on a beginning teacher. *Research in Mathematics Education in Australia*, August 1987, 1-9.

Sullivan, P. (1989). Factors inhibiting change - a case study of a beginning primary teacher. *Mathematics Education Research Journal 1* (2), 1-16.